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Gravitational Spin Entropy Production

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Spin entropy production for particles with arbitrary spin moving in a curved spacetime is discussed. There is a Wigner rotation due to both the acceleration an the curvature, which causes an initial pure state to transform into a final mixed state. Depending on the spacetime characteristics, one may find paths on which there is no Wigner rotation and the state remains pure.

KEY WORDS: local inertial frame; Wigner rotation; density matrix; spin entropy; spherically symmetric spacetimes.

1. INTRODUCTION

Traditionally, Quantum communication theory is based on the entanglement, which is a property unique to quantum systems. This is a strange feature of quantum theory and leads to a nonlocal correlation called the Einstein–Podolsky–Rozen (EPR) correlation (Einstein *et al.*, 1935; Bohm, 1989). Therefore, it is important to study all those processes that might affect quantum entanglement. Recently, a number of papers have discussed how entanglement is affected by the Lorentz transformation in the jargon of special relativity. For instance, Peres *et al.* showed that reduced density matrix for the spin of a single free spin-$\frac{1}{2}$ particle is not covariant under Lorentz boosts, that is, the spin entropy is not a relativistic scalar (Peres *et al.*, 2002). Also, Gingrich and Adami showed that entanglement between two systems depends on the frame in which this entanglement is measured, such that, a fully entangled spin-$\frac{1}{2}$ system loses entanglement if observed by a Lorentz-boosted observer. They discussed, while the entanglement between spin or momentum alone may change due to Lorentz boosts, the entanglement of the entire wave function incorporating spin and momentum is invariant (Gingrich and

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Adami, 2002). More works of this kind can be traced in Alsing and Milborn (2002) and Li and Du (2003).

Recently, the question of how entanglement is affected by a gravitational field is discussed by Terashima and Ueda by extending these considerations to general relativity (Terashima and Ueda, 2004). They have discussed a mechanism of spin decoherence caused by spacetime curvature for spin-$\frac{1}{2}$ particles moving in a gravitational field. In general relativity, a gravitational field represented by spacetime curvature, causes to break the global rotational symmetry. Therefore, the spin in general relativity can be defined only locally by invoking the rotational symmetry of the local inertial frame. Consequently, the motion of the particle in a curved spacetime is accompanied by a continues succession of Lorentz transformations (Terashima and Ueda, 2005). It is shown that this effect gives rise to a spin entropy production that is unique to general relativity. This means that even if the state of the particle is pure at one spacetime point, it becomes mixed at another spacetime point.

In this paper, regarding the work done by Terashima and Ueda, we discuss the gravitational spin entropy production for particles with *arbitrary* spin. We employ a wave packet that its centroid traces a classical path in the curved spacetime. It is assumed that the spacetime curvature does not change extremely within the spacetime scale of the wave packet. We show explicitly that the state of moving particles changes because of successive Lorentz transformations and that there exists a Wigner rotation due to both the acceleration and the curvature that changes the spin part of the state. Therefore, an initial pure state, finally becomes mixed, leading to a production of spin entropy. Then, we apply our general results for static spherically symmetric spacetimes by considering the centroid moving along circular or radial paths.

2. SPIN IN CURVED SPACETIME

As is well known, a gravitational field in general relativity is described by a metric $g_{\mu\nu}(x)$. Here we need to define the spin of a particle in the curved spacetime. Conveniently, we introduce a local inertial frame at each point of spacetime through a tetrad $e_a{}^{\mu}(x)$ defined as

$$e_a{}^{\mu}(x)e_b{}^{\nu}(x)g_{\mu\nu}(x) = \eta_{ab}, \tag{1}$$

where $\eta_{ab} = diag(-1, 1, 1, 1)$ is the Minkowski metric (Nakahara, 1991). This tetrad apparently transforms the general coordinate x^{μ} to a local inertial frame x^{a}. Here and henceforth, it is assumed that Latin indices run over the four inertial-coordinate labels 0, 1, 2, 3 while, Greek letters run over the four general-coordinate labels. The inverse of the tetrad $e_a{}^{\mu}(x)$ is defined as

$$e^a{}_{\mu}(x)e_a{}^{\nu}(x) = \delta_{\mu}{}^{\nu}, \quad e^a{}_{\mu}(x)e_b{}^{\mu}(x) = \delta^a{}_b. \tag{2}$$

Using the tetrad and its inverse, we can transform a tensor given in the general coordinate system into one in the local inertial frame, and vice versa. It must be noted that the choice of the local inertial frames is not unique, since the inertial frame remains inertial under the local Lorentz transformation. More precisely, definitions (1) and (2) remain intact under the transformation

$$e'_a{}^{\mu}(x) = \Lambda_a{}^b(x) e_b{}^{\mu}(x), \quad e'^a{}_{\mu}(x) = \Lambda^a{}_b(x) e^b{}_{\mu}(x). \tag{3}$$

where $\Lambda_a{}^b(x)$ denotes the local Lorentz transformation including spatial rotations and boosts. Here, a particle is specified by the tetrad $e_0{}^{\mu}$, which relates the local time to a global time. Of course, since we cannot uniquely choose the time coordinate to define the positive energy, the definition of a particle is not unique (Birrel and Davies, 1982). Now, a particle with spin s in the curved spacetime is defined as a particle whose one-particle states furnish the spin-s representation of the local Lorentz transformation. In other words, the spin angular momentum components in the local inertial frame are just the generators of the local rotation group.

Consider that the centroid of a wave packet of a massive spin-s particle is located at point x^{μ} and is moving with four-velocity $u^{\mu}(x) = dx^{\mu}/d\tau$, that is normalized as $u^{\mu}(x)u_{\nu}(x) = -c^2$. Let M be the mass of the particle. The four-momentum of the centroid then becomes $q^a(x) = e^a{}_{\mu}(x)\,(Mu^{\mu}(x))$ in the local inertial frame at the point x^{μ}. Using this local inertial frame, we can describe the wave packet as in the special relativity. It is required to assume that the spacetime curvature does not change drastically within the spacetime scale of the wave packet that describes a state of the particle (Terashima and Ueda, 2005). In the local inertial frame, the one-particle momentum eigenstate is described as $|p^a, m\rangle$ where $p^a = (\sqrt{|\mathbf{p}|^2 + m^2c^2}, \mathbf{p})$ is the four momentum of the particle and m denotes the z-components of the spin, such that $-s \leq m \leq s$. Then a wave packet with *positive energy*, has the form

$$|\psi\rangle = \sum_m \int d^3p \left(\frac{Mc}{p^0}\right) C(p^a, m)|p^a, m\rangle, \tag{4}$$

where $d^3p\left(\frac{Mc}{p^0}\right)$ is a Lorentz-invariant volume element and the amplitudes $C(p^a, m)$ determine the admixture of the one particle momentum eigenstates in the wave packet. Normalizing $|\psi\rangle$ to unity implies

$$\sum_m \int d^3p \left(\frac{Mc}{p^0}\right) |C(p^a, m)|^2 = 1, \tag{5}$$

provided that $\langle p'^a, m'|p^a, m\rangle = \left(\frac{p^0}{Mc}\right)\delta^3(\mathbf{p}' - \mathbf{p})\delta_{m'm}$. Corresponding to the wave packet (4), there exists a density matrix $\rho = |\psi\rangle\langle\psi|$, which its trace over the momentum leads to the reduced density matrix for the spin, that is

$$(\varrho)_{m'm} = \int d^3p \left(\frac{Mc}{p^0}\right) C(p^a, m')C^*(p^a, m). \tag{6}$$

Then the spin entropy of the wave packet at every point x^μ is the von Newman entropy of this reduced density matrix, defined as

$$S(x) = -\mathrm{Tr}\left[\varrho \log_2 \varrho\right]. \tag{7}$$

After an infinitesimal proper time $d\tau$, the centroid moves to a new point $x'^\mu = x^\mu + u^\mu(x)d\tau$, and the wave packet is then described by the local inertial frame at the new point x'^μ. In the new local inertial frame, the momentum of the centroid changes to $q^a(x') = q^a(x) + \delta q^a(x)$. The explicit form of $\delta q^a(x)$ is given by Terashima and Ueda (2004)

$$\delta q^a(x) = \lambda^a{}_b(x)q^b(x)d\tau, \tag{8}$$

with

$$\lambda^a{}_b(x) = -\frac{1}{mc^2}[a^a(x)q_b(x) - q^a(x)a_b(x)] + \chi^a{}_b(x), \tag{9}$$

where

$$a^a(x) = e^a{}_\mu(x)\big(u^\nu(x)\nabla_\nu u^\mu(x)\big), \tag{10}$$

is the acceleration caused by the external force, and

$$\chi^a{}_b(x) = u^\mu(x)\big(e_b{}^\nu(x)\nabla_\mu e^a{}_\nu(x)\big), \tag{11}$$

is the change in the local inertial frame along $u^\mu(x)$.

The first term in (9) exists even in special relativity. While, the second term due to the spacetime curvature exists only in general relativity. For geodesic motions of the centroid, $a^a(x) = 0$ and the first term vanishes. It must be noted that $\lambda^a{}_b$ given by (9) satisfies the condition $\lambda_{ab} = -\lambda_{ba}$, such that the infinitesimal local Lorentz transformation can be written as as $\Lambda^a{}_b(x) = \delta^a{}_b + \lambda^a{}_b(x)d\tau$. Corresponding to the local LT $\Lambda^a{}_b(x)$, there exists a unitary operator, denoted by $U(\Lambda^a{}_b(x))$, which transforms the momentum eigenstate as

$$|p^a, m\rangle \to U(\Lambda(x))|p^a, m\rangle. \tag{12}$$

It can be shown that the state $U(\Lambda)|p^a, m\rangle$ is also the eigenstate of the momentum operator but with the eigenvalue Λp and we can write at every point (Weinberg, 1995)

$$U(\Lambda(x)|p^a, m\rangle = \sum_{m'} D^{(s)}_{m'm}(W(\Lambda(x), p))|\Lambda p^a, m'\rangle, \tag{13}$$

where $W^a{}_b(\Lambda(x), p)$ is the Wigner rotation and $D^{(s)}_{m'm}(W(\Lambda(x), p))$ is its unitary representation. The infinitesimal Wigner rotation is represented as

$$W^a{}_b(\Lambda(x), p) = \delta^a{}_b + \vartheta^a{}_b d\tau, \tag{14}$$

where $\vartheta^0{}_0(x) = \vartheta^0{}_i(x) = \vartheta^i{}_0(x) = 0$ and

$$\vartheta^i{}_k(x) = \lambda^i{}_k(x) + \frac{\lambda^i{}_0(x)p_k(x) - \lambda_{k0}(x)p^i(x)}{p^0(x) + Mc}, \tag{15}$$

where i and k run over the three spatial inertial frame labels (1, 2, 3). Note that $\vartheta_{ii} = 0$. The infinitesimal Wigner rotation (14) has correspondingly a spin-s irreducible representation as

$$\begin{aligned} D^{(s)}_{m'm}(W(\Lambda(x), p)) = \delta_{m'm} &+ i\vartheta_{23}(x)\big(J_1^{(s)}\big)_{m'm}d\tau + i\vartheta_{31}(x)\big(J_2^{(s)}\big)_{m'm}d\tau \\ &+ i\vartheta_{12}(x)\big(J_3^{(s)}\big)_{m'm}d\tau, \end{aligned} \tag{16}$$

where $\{J_1^{(s)}, J_2^{(s)}, J_3^{(s)}\}$ are the generators of the rotation group in a $(2s+1)$-dimensional representation (Weinberg, 1995).

When the centroid moves along a path $x^\mu(\tau)$ from $x_i^\mu = x^\mu(\tau_i)$ to $x_f^\mu = x^\mu(\tau_f)$, the motion of the wave packet is accompanied by a LT given by Terashima and Ueda (2005)

$$\Lambda(x_f, x_i) = T \exp\left[\int_{x_i}^{x_f} \lambda\,(x(\tau))\,d\tau\right], \tag{17}$$

and, a Wigner rotation as

$$W(\Lambda(x_f, x_i), p) = T \exp\left[\int_{x_i}^{x_f} w\,(x(\tau))\,d\tau\right], \tag{18}$$

where T is the time-ordering operator and λ and w are matrices that their components are given by (9) and (15), respectively.

Suppose that in the local inertial frame at the initial point x_i^μ, the wave packet is denoted as $|\psi^{(i)}\rangle$ as given by (4), with corresponding reduced density matrix $\varrho^{(i)}_{m'm}$ as given by (6). Then, in the local inertial frame at the final point x_f^μ, the wave packet will be

$$\begin{aligned} |\psi^{(f)}\rangle = U(\Lambda(x_f, x_i))|\psi^{(i)}\rangle = \sum_m \sum_{m'} \int d\mathbf{p} \left(\frac{Mc}{p^0}\right) C(p^a, m) \\ \times D^{(s)}_{m'm}(W(\Lambda(x_f, x_i), p))|\Lambda p, m'\rangle, \end{aligned} \tag{19}$$

which leads to a reduced density matrix as

$$\varrho^{(f)}_{m'm} = \sum_{m''m'''} \int d\mathbf{p} \left(\frac{Mc}{p^0}\right) C(p^a, m'')C^*(p^a, m''')$$
$$\times D^{(s)}_{m'm''}(W(\Lambda(x_f, x_i), p))D^{(s)*}_{mm'''}(W(\Lambda(x_f, x_i), p)). \tag{20}$$

For instance, assume that at the initial proper time τ_i, the coefficient of expansion in (4) is written as $C(p^a, m) = F(p^a)\delta_{m,s}$ where $F(p^a)$ is a normalized function having a significant peak at the four-momentum of the centroid q^a. Substituting this in (6), we obtain the initial reduced density matrix as

$$\varrho^{(i)} = \begin{pmatrix} 1 & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \tag{21}$$

which apparently describes a pure state with zero spin entropy. While, the final reduced density matrix (20) becomes

$$\varrho^{(f)} = \begin{pmatrix} \overline{|D^{(s)}_{s,s}|^2} & \overline{D^{(s)}_{s,s}D^{(s)*}_{s-1,s}} & \cdots & \overline{D^{(s)}_{s,s}D^{(s)*}_{-s,s}} \\ \overline{D^{(s)}_{s-1,s}D^{(s)*}_{s,s}} & \overline{|D^{(s)}_{s-1,s}|^2} & \cdots & \overline{D^{(s)}_{s-1,s}D^{(s)*}_{-s,s}} \\ \vdots & \vdots & \ddots & \vdots \\ \overline{D^{(s)}_{-s,s}D^{(s)*}_{s,s}} & \overline{D^{(s)}_{-s,s}D^{(s)*}_{s-1,s}} & \cdots & \overline{|D^{(s)}_{-s,s}|^2} \end{pmatrix}, \tag{22}$$

where overline is defined as $\overline{X} = \int d\mathbf{p}N(p^a)|F(p^a)|^2 X(p^a)$ which denotes the average over the momentum distribution. The state described by (22) is generally mixed and there exists a spin entropy, generated by the gravity and acceleration, given by

$$S^{(f)} = -\sum_{i=1}^{2s+1} d^{(s)}_i \log_2 d^{(s)}_i, \tag{23}$$

where $d^{(s)}_i$ denotes the i-th eigenvalue of $\varrho^{(f)}$. Consequently, even if the initial state is pure, after moving in the curved spacetime, the final state will be mixed.

3. SPIN ENTROPY PRODUCTION IN A STATIC SPHERICALLY SYMMETRIC SPACETIME

Here we consider the gravitation field to be a static spherically symmetric described as

$$ds^2 = -c^2 e^{2A(r)} dt^2 + e^{2B(r)} dr^2 + r^2 \left(d\theta^2 + \sin^2\theta\, d\phi^2\right) \tag{24}$$

where $A(r)$ and$B(r)$ are two functions to be determined. Let the metric be asymptotically flat. This assumption imposes the following conditions on $A(r)$ and $B(r)$,

$$\lim_{r\to\infty} A(r) = \lim_{r\to\infty} B(r) = 0. \tag{25}$$

It must be noted that since the metric (24) is static, the time coordinate t is a Killing time.

We require to introduce a static observer with a static local inertial frame at each point in the spacetime (24). Therefore, we choose the tetrad defined in (1) as

$$e_0{}^t = \frac{1}{c} e^{-A(r)}, \quad e_1{}^r = e^{-B(r)}, \quad e_2{}^\theta = \frac{1}{r}, \quad e_3{}^\phi = \frac{1}{r\sin\theta}, \tag{26}$$

with all the other components being zero. By this choice, the local inertial time coordinate x^0 is parallel to the Killing time coordinate t.

In the following, we will consider spin-s particles moving in a spacetime described by (24). We will argue on circular and radial motions, separately.

3.1. Circular Motion Around the Center

Suppose that the centroid of the wave packet of a particle with spin-s is moving with a constant speed v on a circle with radius r around the center. Regarding the spherical symmetry of the metric, we can choose the plane of motion to be the equatorial plane $\theta = \frac{\pi}{2}$. If v is measured by an observer in the local inertial frame, who uses inertial coordinate labels (0, 1, 2, 3), we can write

$$v = c\frac{d\,\hat{3}}{d\,\hat{0}} = c\frac{e^3{}_\phi\, d\phi}{e^0{}_t\, dt} = re^{-A(r)}\frac{d\phi}{dt}. \tag{27}$$

Then, we obtain the non-zero components of the four-velocity as

$$u^t = e^{-A(r)}\cosh\xi, \quad u^\phi = \frac{c}{r}\sinh\xi, \tag{28}$$

where $\xi = \tanh^{-1}(\frac{v}{c})$. Accordingly, the components of the four-momentum of the centroid in the local inertial frame at any point are

$$q^0 = Mc\cosh\xi, \quad q^1 = q^2 = 0, \quad q^3 = Mc\sinh\xi, \tag{29}$$

which are constant. Moreover, after some manipulation we see that in this case, the acceleration (10) have only a non-zero component as

$$a^1(r) = c^2 e^{-B(r)} \left(A'(r)\cosh^2\xi - \frac{1}{r}\sinh^2\xi \right). \tag{30}$$

where prime denotes the differentiation with respect to r. This acceleration gives rise to a generalized Thomas precession, via the first term of (9).

Now, regarding (11) and after doing some manipulation, we obtain in this case the non-zero components of $\chi^a{}_b(x)$ as

$$\chi^0{}_1(r) = \chi^1{}_0(x) = -c\, A'(r) e^{-B(r)} \cosh\xi, \tag{31}$$

which consists of a boost along the 1-axis, and

$$\chi^1{}_3(r) = -\chi^3{}_1(r) = c\, \frac{e^{-B(r)}}{r} \sinh\xi \tag{32}$$

which generates spatial rotations about the 2-axis.

Then, substituting these results in (9), we see that $\lambda^a{}_b(x)$ has only four non-zero components as

$$\lambda^1{}_0(r) = \lambda^0{}_1(r) = \left(A'(r) - \frac{1}{r} \right) c\, e^{-B(r)} \sinh^2\xi \cosh\xi, \tag{33}$$

$$\lambda^1{}_3(r) = -\lambda^3{}_1(r) = -\left(A'(r) - \frac{1}{r} \right) c\, e^{-B(r)} \sinh\xi \cosh^2\xi, \tag{34}$$

which lead to the following non-zero components for $\vartheta^a{}_b$

$$\vartheta^1{}_3(p^a) = -\vartheta^3{}_1(p^a) = c \left(A'(r) - \frac{1}{r} \right) e^{-B(r)} \left(\frac{p^3}{p^0 + Mc} - 1 \right) \sinh\xi \cosh^2\xi \tag{35}$$

where we have used (29). Then the corresponding infinitesimal Wigner rotation (14), becomes

$$D^{(s)}_{m'm}(W(\Lambda), p) = \delta_{m'm} + i\vartheta(p^a)\big(J_2^{(s)}\big)_{m'm} d\tau \tag{36}$$

where

$$\vartheta(p^a) = \vartheta_{31}(p^a) = \frac{q^3 q^0}{M^3 c^2} e^{-B(r)} \left(A'(r) - \frac{1}{r} \right) \left(\frac{q^3 p^3}{p^0 + Mc} - q^0 \right). \tag{37}$$

Since $\vartheta(p^a)$ is independent of time, we do not need to apply the time ordering operator and we can simply generalize the infinitesimal Wigner rotation (36) for a finite time interval $\tau = \tau_f - \tau_i$, that is

$$D^{(s)}(W(\Lambda(\tau), p)) = \exp\left[i J_2^{(s)} \vartheta(p^a)\tau \right], \tag{38}$$

which represents a rotation about 2-axis with an angle of rotation $\vartheta(p^a)\tau$. The elements of (38) can be obtained generally by the Wigner's formula, that is (Sakurai, 1985)

$$D^{(s)}_{m'm}(W(\Lambda(\tau), p)) = \sum_k (-1)^k \frac{\sqrt{(s+m)!(s-m)!(s+m')!(s-m')!}}{(s+m-k)!k!(s-k-m')!(k-m+m')!} \times \left(\cos\frac{\vartheta(p^a)\tau}{2}\right)^{2s-2k+m-m'} \left(\sin\frac{\vartheta(p^a)\tau}{2}\right)^{2k-m+m'} \tag{39}$$

where the sum over k is taken whenever none the arguments of factorials are negative. Substituting these elements in (22), we can determine the final density matrix for a given s. Of course, this matrix will be too sophisticated to be used. Here, as two simple examples we give the results for $s = \frac{1}{2}$ and $s = 1$ cases. Choosing $s = \frac{1}{2}$ in (39), we obtain

$$\varrho^{(f)} = \frac{1}{2}\begin{pmatrix} 1 + \overline{\cos\vartheta\tau} & -\overline{\sin\vartheta\tau} \\ -\overline{\sin\vartheta\tau} & 1 - \overline{\cos\vartheta\tau} \end{pmatrix}. \tag{40}$$

While, for $s = 1$ the final density $\varrho^{(f)}$ becomes

$$\begin{pmatrix} \frac{1}{4}\overline{(1+\cos\vartheta\tau)^2} & -\frac{\sqrt{2}}{4}\overline{(1+\cos\vartheta\tau)\sin\vartheta\tau} & \frac{1}{4}\overline{\sin^2\vartheta\tau} \\ -\frac{\sqrt{2}}{4}\overline{(1+\cos\vartheta\tau)\sin\vartheta\tau} & \frac{1}{2}\overline{\sin^2\vartheta\tau} & -\frac{\sqrt{2}}{4}\overline{(1-\cos\vartheta\tau)\sin\vartheta\tau} \\ \frac{1}{4}\overline{\sin^2\vartheta\tau} & -\frac{\sqrt{2}}{4}\overline{(1-\cos\vartheta\tau)\sin\vartheta\tau} & \frac{1}{4}\overline{(1-\cos\vartheta\tau)^2} \end{pmatrix}$$

One may argue that, according to (37) if $A(r)$ satisfies the condition $rA'(r) = 1$, $\vartheta(p^a)$ will vanish and therefore the final density matrix (22) becomes identical with the initial pure density matrix (21). But, solving this condition, we obtain $A(r) \sim \ln r$, which leads to $g_{00} \sim r$. This apparently violates the asymptotic flatness of the metric (24) and here we do not pursue this argument.

3.2. Radial Motion

Now let us consider that the centroid of the wave packet moves with a velocity v along a radial geodesic. Since v is measured by an observer in the local inertial frame, we can write

$$v = c\,\frac{d\,\hat{1}}{d\,\hat{0}} = c\,\frac{e^1{}_r\,dr}{e^0{}_t\,dt} = e^{B(r)}e^{-A(r)}\frac{dr}{dt}. \tag{41}$$

Then, we obtain in this case the non-zero components of the four-velocity (in the general frame) as

$$u^t = e^{-A(r)}\cosh\xi, \quad u^r = c\,e^{-B(r)}\sinh\xi, \tag{42}$$

which lead to

$$q^0 = Mc \cosh\xi, \quad q^1 = Mc \sinh\xi, \quad q^2 = q^3 = 0, \tag{43}$$

as the components of the four-momentum of the centroid.

Since the centroid moves along a geodesic, $a^a(x) = 0$ and then the acceleration related part in (9) vanishes. However, after doing some manipulations we see that the curvature related part has two non-zero components as

$$\chi^0{}_1(r) = \chi^1{}_0(x) = -c\, A'(r)e^{-B(r)} \cosh\xi. \tag{44}$$

Thus, the local infinitesimal LT has only two non-zero components as

$$\lambda^0{}_1(r) = \lambda^1{}_0(r) = -c\, A'(r)e^{-B(r)} \cosh\xi, \tag{45}$$

which consists of a boost along the 1-axis.

Applying (45) in (15), we conclude that all of the components of $\vartheta^i{}_k$ vanish and the Wigner rotation simply becomes $W^a{}_b = \delta^a{}_b$, which has a trivial spin-s representation as $D^{(s)}_{m'm}(W(x)) = \delta_{m'm}$. These elements as substituted in (22), give a pure density matrix identical with (21). Thus, there is no spin entropy production in the case of radial motion.

However, there exists still a local LT as the centroid moves from r_i to r_f, that is

$$\Lambda(r_f, r_i) = T \exp\left(\int_{r_i}^{r_f} \lambda\,(r(\tau))\, d\tau\right). \tag{46}$$

where the elements of the matrix $\lambda(r)$ are given by (45). Correspondingly, the operator $U(\Lambda^a{}_b(r_f, r_i))$ will transform the initial state of radially falling particles into a boosted frame along the 1-axis. One must be careful of time ordering, since in the radial motion the variables r and τ are not independent.

4. DISCUSSION

We have generally proved that both acceleration and gravity cause to produce spin entropy for particles moving along a path in a curved spacetime. For circular paths in the static spherically symmetric spacetimes, we obtained an explicit expression for the Wigner rotation, and determined the final mixed density matrix. In this case as (37) exhibits, far from the center both gravity and acceleration vanish, leading to a final pure density matrix with zero spin entropy. On the other hand, depending on the nature of the spacetime we may find a circular path on it the r dependent part of (37), that is $\Gamma(r) = e^{-B(r)}[A'(r) - 1/r]$, vanishes. Indeed, on such a path the gravitational and the accelerative effects cancel each other, and so the final state will be pure with zero spin entropy. To illustrate the points, it is convenient to consider some examples.

First we consider a black hole for that $e^{-2B(r)} = e^{2A(r)} = (1 - r_s/r)$, with r_s being the Schwarzschild radius. Then we have $\Gamma(r) = (1 - r_s/r)^{-1/2}(1 - 3r_s/2r)$, which diverges at the event horizon, leading to a maximum mixing of the final state. While, it vanishes for paths on the sphere of radius $r = 3r_s/2$, leading to a zero spin entropy. Because of peculiarity of the event horizon, there is a discrepancy in the results of subsection 3.2 for radial paths terminating to points inside the surface of the event horizon. One can make a similar but elaborate argument for charged black holes.

Alternatively, for a *traversable* wormhole (Morris and Thorne, 1988), we have $e^{-2B(r)} = 1 - b(r)/r$ with $A(r)$ being a finite function everywhere. In wormhole physics $b(r)$ and $A(r)$ are called shape-function and redshift function, respectively. The finiteness of $A(r)$ guaranties that the metric (24) has no event horizon. The equation $b(r) = r$ determines the radius of the throat of wormhole that is the minimum value of r. In this case we have, $\Gamma(r) = (1 - b(r)/r)[A'(r) - 1/r]$, which is finite everywhere and vanishes only just at the throat. Then, for circular paths coinciding the throat, the initial pure state remains intact. Because of the absence of the event horizon, wave packets moving along a radial path, hence without any spin entropy production, can traverse the throat and reach to the other side of wormhole. This last point can be important for anyone who is going to study information transferring through wormholes.

REFERENCES

Alsing, P. M. and Milborn, G. J. (2002). *Quantum Information and Computation* **2**, 487.

Birrel, N. D. and Davies, P. C. W. (1982). *Quantum Fields in Curved Spaces*, Cambridge University press, Cambridge.

Bohm, D. (1989). *Quantum Theory*, Dover, New York.

Einstein, A., Podolsky, B., and Rozen, N. (1935). *Physical Review* **47**, 777.

Gingrich, R. M. and Adami, C. (2002). *Physical Review Letters* **89**, 270402.

Li, H. and Du, J. (2003). *Physical Review A* **68**, 022108.

Morris, M. S. and Thorne, K. S. (1988). *American Journal of Physics* **56**, 395.

Nakahara, M. (1991). *Geometry, Topology and Physics*, Adam Hilger.

Peres, A., Scudo, P. F., and Terno, D. R. (2002). *Physical Review Letters* **88**, 230402.

Terashima, H. and Ueda, M. (2004). *Physical Review A* **69**, 032113.

Terashima, H. and Ueda, M. (2005). *Journal of Physics A*, 2029.

Sakurai, J. J. (1985). *Modern Quantum Mechanics*, Addison-Wesley, New York.

Weinberg, S. (1995). *The Quantum Theory of Fields*, Cambridge University Press, Cambridge.